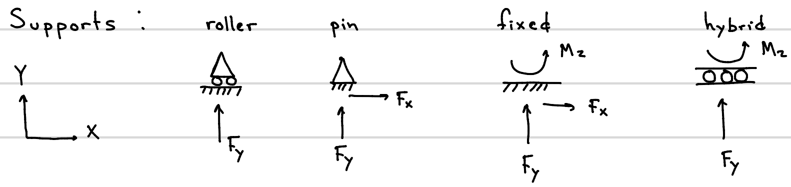
January 15, 2026
Statics Review
Part I - Equilibrium

Objectives :

1. being able to identify various types of supports and associated reactions
2. calculating external reactions (internal reaction) for statically determinate structures

Supports :



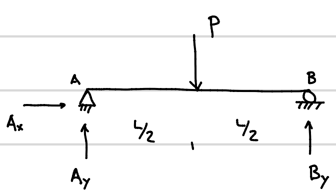
All correct solutions to mechanics problems must satisfy three principles :

1. Equilibrium - balance of forces/moments
2. Compatibility - support conditions are satisfied, structural elements don't develop kinks or discontinuities
3. Constitutive behavior - stress/strain : force/displacement relationships

Static Equilibrium $\Sigma F_i = 0$ in 2D - $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$ (3 total equations)

in 3D - $\Sigma F_{x,y,z} = 0$, $\Sigma M_{x,y,z} = 0$ (6 total equations)

EX 1.

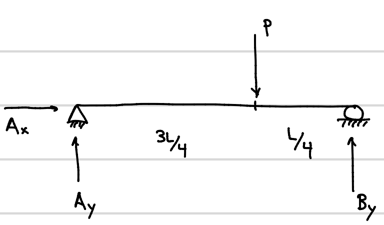


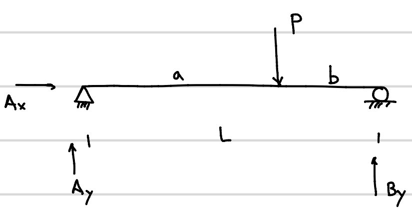
statically determinate

$\xrightarrow{+}$ $\Sigma F_x = 0$ $\quad A_x = 0$

$+\uparrow$ $\Sigma F_y = 0$ $\quad A_y + B_y - P = 0 \quad A_y + B_y = P \quad \therefore \quad A_y = P/2$

$\overset{+}{\circlearrowleft}$ $\Sigma M_A = 0$ $\quad B_y \cdot L - P \cdot L/2 = 0 \quad B_y = P/2$



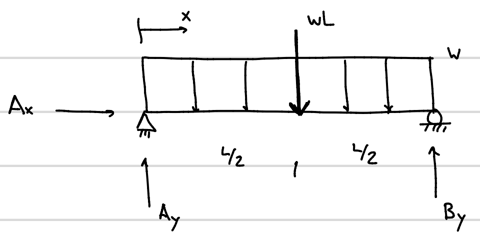


$\Sigma F_y = 0 \quad A_y + B_y = P \qquad A_y = P/4$

$\Sigma M_A = 0 \quad B_y \cdot L - P \cdot 3L/4 = 0 \qquad B_y = 3P/4$

$A_y = Pb/L \qquad B_y = Pa/L$

EX 2.



resultant force : magnitude = area

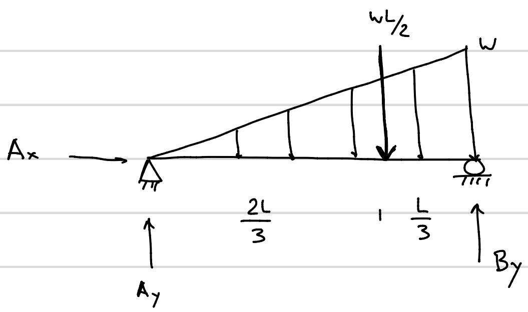
location = centroid

$$\text{Area } (A) = \int dA \qquad \int dA = \iint dy\,dx = \int_0^L \int_0^w dy\,dx = wL$$

$$\text{Centroid } (\bar{x}) = \frac{\int x\,dA}{\int dA} \qquad \int x\,dA = \int_0^L \int_0^w x\,dy\,dx = \int_0^L \Big|_0^w xy\,dx = w\int_0^L x\,dx = w\Big|_0^L \frac{x^2}{2} = \frac{wL^2}{2}$$

$$\bar{x} = \frac{\frac{wL^2}{2}}{wL} = L/2$$

EX. 3



$$\text{Area of triangle} = \tfrac{1}{2}\,b\,h = \tfrac{1}{2}(L)(w) = \frac{wL}{2}$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} \qquad \int dA \rightarrow \frac{wL}{2}$$

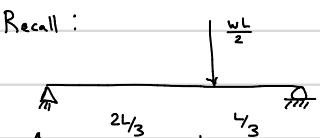
$$\int x\,dA = \int_0^L \int_0^{wx/L} x\,dy\,dx = \int_0^L \Big|_0^{wx/L} y\,x\,dx = \int_0^L \frac{wx^2}{L}\,dx = \frac{w}{L}\int_0^L x^2\,dx = \frac{w}{L}\Big|_0^L \frac{x^3}{3} = \frac{wL^3}{3L} = \frac{wL^2}{3}$$

$$\bar{x} = \frac{\frac{wL^2}{3}}{\frac{wL}{2}} = \frac{2L}{3}$$

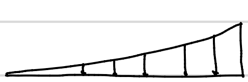
$$\Sigma F_y = 0 \quad A_y + B_y - \frac{wL}{2} = 0 \qquad A_y = wL/6$$

$$\Sigma M_A = 0 \quad B_y \cdot L - \frac{wL}{2}\cdot\frac{2L}{3} = 0 \qquad B_y = wL/3$$

Recall :



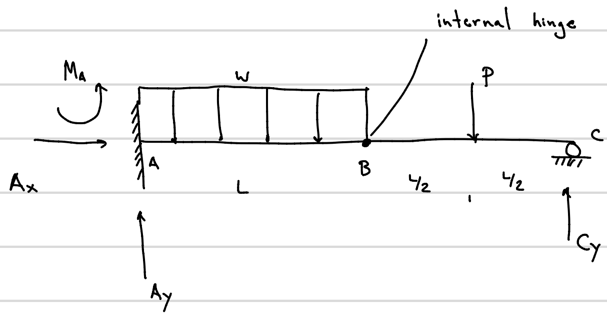
$$A_y = \frac{\frac{wL}{2}\cdot\frac{L}{3}}{L} = \frac{wL}{6} \qquad B_y = \frac{\frac{wL}{2}\cdot\frac{2L}{3}}{L} = \frac{wL}{3}$$



$q(x)$ generalized loading

$$A = \int dA = \int_0^L \int_0^{q(x)} dy\,dx \qquad \bar{x} = \frac{\int x\,dA}{\int dA}$$

Ex. 4.

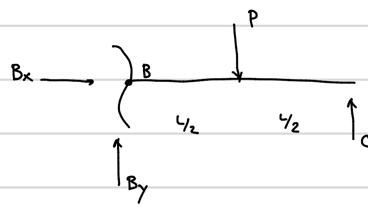


4 unknowns

3 equilibrium equations (overall)

+ 1 hinge equilibrium equation $M_B = 0$ (allows rotation)

$\therefore$ statically determinate

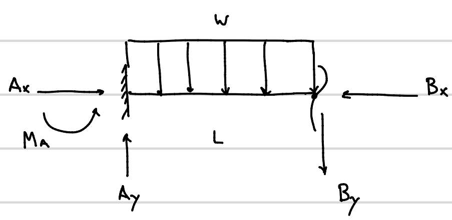


$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma F_y = 0 \quad B_y + C_y - P = 0 \qquad B_y = P/2$

$\Sigma M_B = 0 \quad C_y * L - P * L/2 = 0 \qquad C_y = P/2$

<u>Option 1</u> : solve left-portion

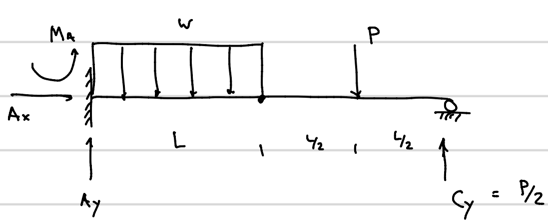


Newton's 3rd Law

Equal + Opposite

↓ magnitude ↓ direction (vector)

<u>Option 2</u> : solve entire structure



$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad A_y - w * L - P + C_y = 0 \qquad A_y = wL + P/2$

$\Sigma M_A = 0 \quad C_y * 2L - P * 3L/2 - wL * L/2 + M_A = 0 \qquad M_A = \frac{wL^2}{2} + \frac{PL}{2}$