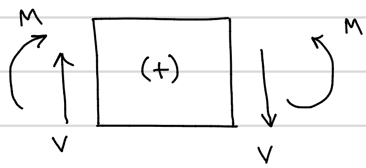
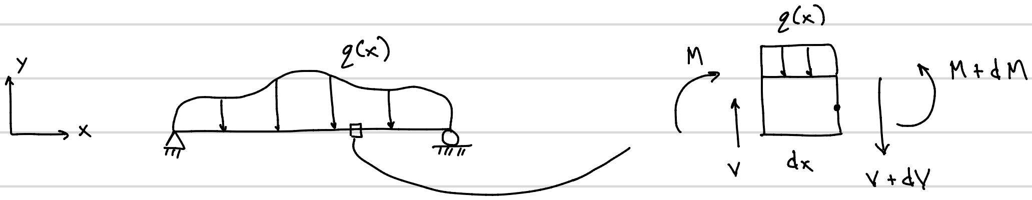
January 22, 2026
Solids Review
Part I - Shear/Moment

Shear and Moment Diagrams

Sign convention





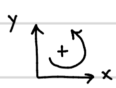
$$\sum F_y = 0 \qquad V - (V + dV) - q(x)\,dx = 0$$

$$-dV = q(x)\,dx$$

$$\boxed{\frac{dV}{dx} = -q(x)}$$

$$dV = -q(x)\,dx \qquad \int_A^B dV = \int_A^B -q(x)\,dx \qquad \therefore \; V_B = V_A - \text{area under } q(x) \; *$$

$$\sum M_o = 0 \qquad -V\,dx - M + (M + dM) + \overbrace{q(x)\,dx\,\frac{dx}{2}}^{\text{higher order term} \sim dx^2 \to \emptyset} = 0$$

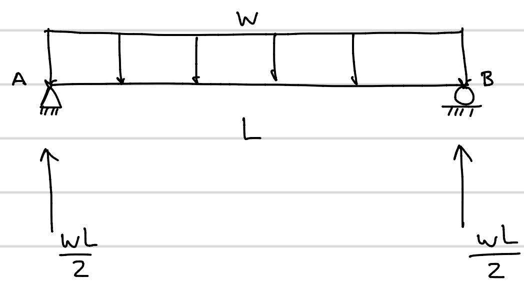


$$dM = V\,dx$$

$$\boxed{\frac{dM}{dx} = V}$$

$$dM = V\,dx \qquad \int_A^B dM = \int_A^B V\,dx \qquad \therefore \; M_B = M_A + \text{area under } V(x) \; *$$

Ex. 1

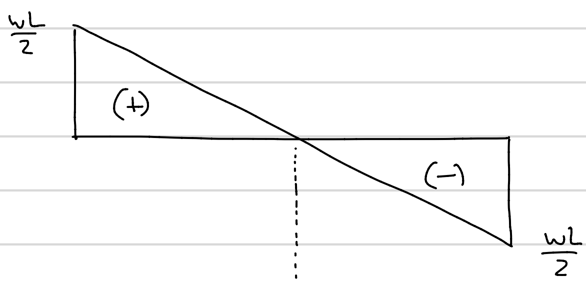


$q(x)$ loading : constant x^0

$V(x)$ shear : linear x^1

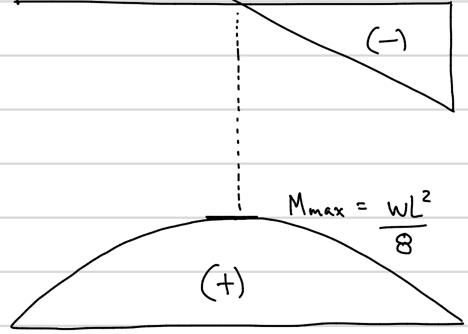
$M(x)$ moment : quadratic x^2

$V(x)$



local max/min occurs where slope = 0 !

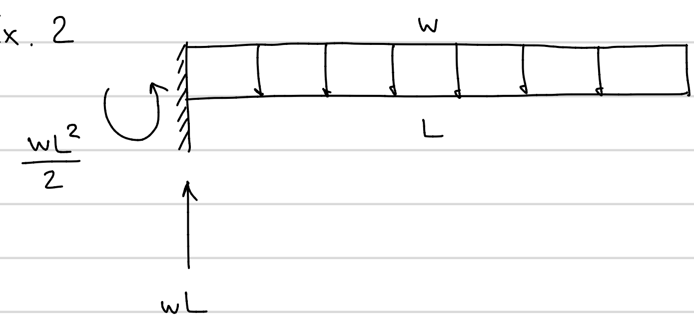
$M(x)$

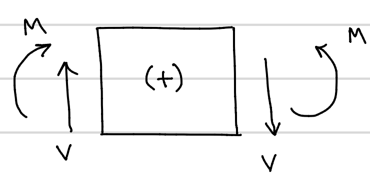


$M_{max} = M_A$ + area under left $V(x)$

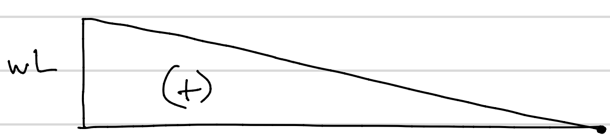
$= 0 + \frac{1}{2}\left(L/2\right)\left(wL/2\right)$

Ex. 2



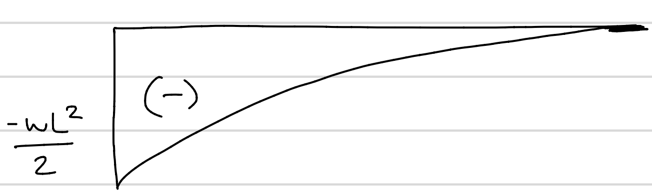


$V(x)$



$V(x) = 0$ at free end

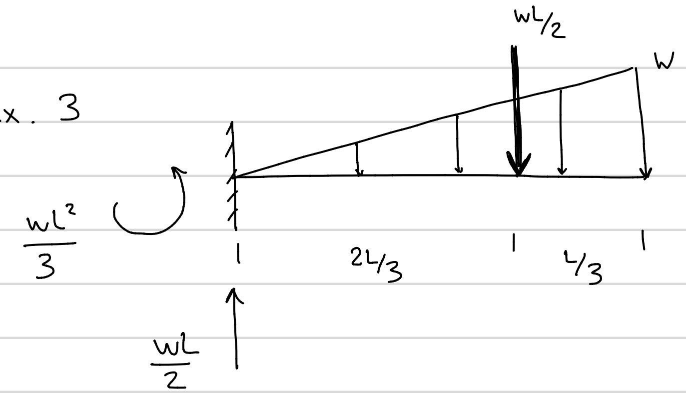
$M(x)$



$$\frac{dM(x)}{dx} = V(x) = 0$$

slope!

Ex. 3

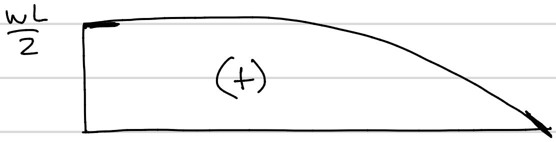


$q(x)$ = linear

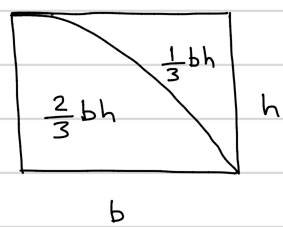
$V(x)$ = quadratic

$M(x)$ = cubic

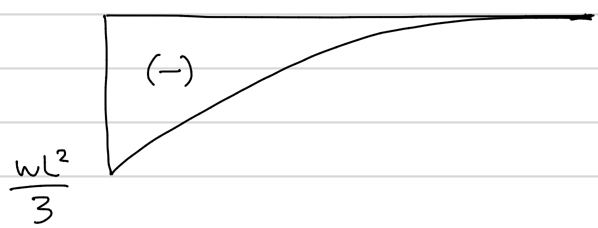
$V(x)$



$$\frac{dV}{dx} = -w$$

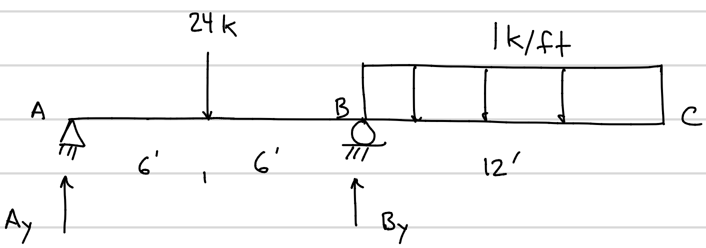


$M(x)$



$$\frac{dM}{dx} = 0$$

Ex. 4



1. find reactions

$$\Sigma F_y = 0 \quad A_y + B_y - 24 - 1(12) = 0$$

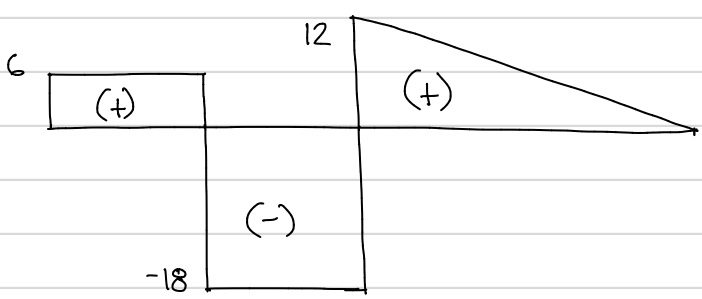
$$A_y + B_y = 36$$

$$\Sigma M_A = 0 \quad B_y(12) - 24(6) - 12(18) = 0$$

$$B_y = 30 \quad \therefore \quad A_y = 6$$

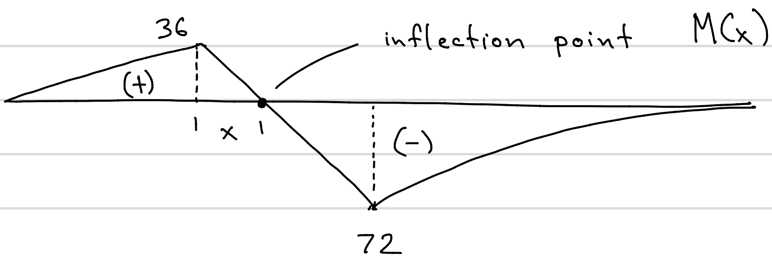
$V(x)$

k

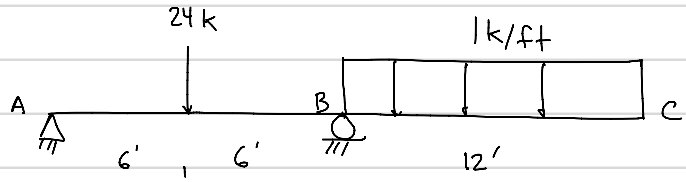


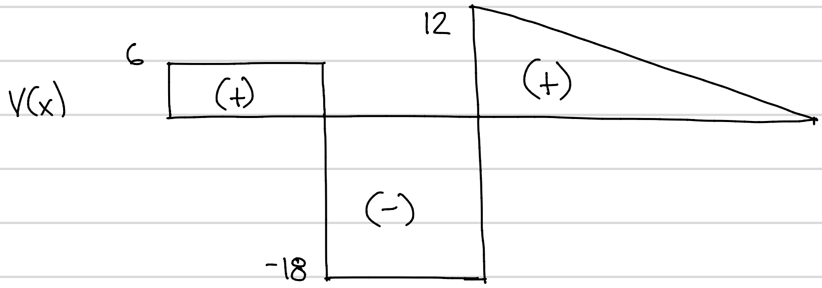
$M(x)$

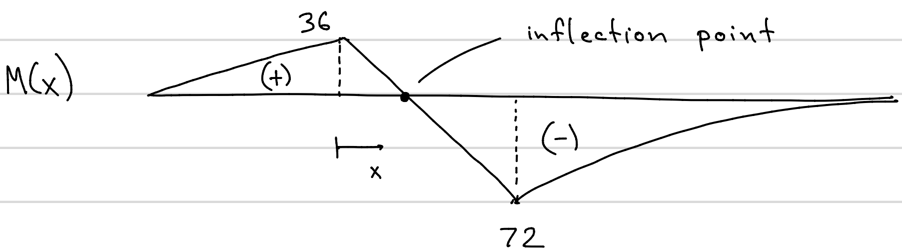
k·ft



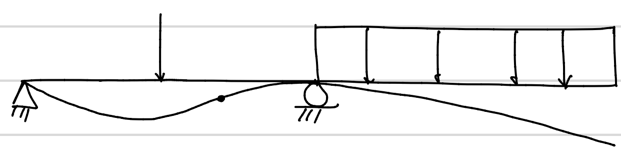
$M(x) = 0$







sign of M(x) = concavity



Inflection Point Location

1) from similar triangles $\frac{36}{x} = \frac{72}{6-x}$ $x = 2'$ * (8' from point A) *

2) from line slope, i.e. $\frac{dM}{dx} = V$ $\frac{36}{18} = 2'$ *

3) from equations $6 \leq x < 12$ (three different regions)

$V(x) = -18$

$M(x) = \int V(x) = -18x + C$ @ $x = 6$ $M(x) = -72$ $\therefore$ $C = 36$

$M(x) = -18x + 36$ $-18x + 36 = 0$ $\therefore$ $x = 2'$ *