January 27, 2026
Solids Review
Part II - Stress/Deflection

Euler-Bernoulli Beam Theory (EBBT)

Assumptions: slender, prismatic, vertical axis of symmetry, initially straight, isotropic, homogeneous
* displacements are small enough such that equilibrium equations can be written about original (undeformed) configuration

Thus: the state of stress is uniaxial, plane sections remain plane

Normal (bending) stress: $\sigma_b = -\frac{My}{I}$ M = bending moment

y = distance from centroid

I = moment of Inertia (2nd moment of area)

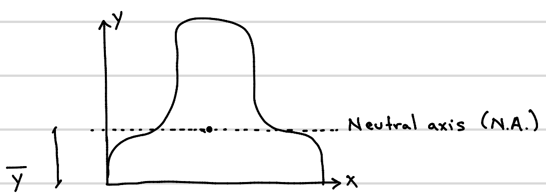
Shear stress ($L/d \geq 10 \therefore$ EBBT is valid)

$\tau = \frac{VQ}{Ib}$ V = shear force

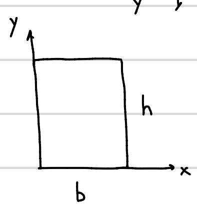
Q = 1st moment of area $\int y\,dA$ * about neutral axis

I = moment of inertia $\int y^2\,dA$ * about neutral axis

b = width

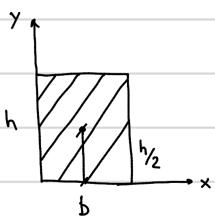


$$\bar{y} = \frac{\int y\,dA}{\int dA} \quad \text{— Area}$$



Area = bh

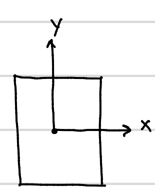
$$\int y\,dA = \int_0^b \int_0^h y\,dy\,dx = \int_0^b \frac{h^2}{2}\,dx = \frac{bh^2}{2} \qquad \bar{y} = \frac{\frac{bh^2}{2}}{bh} = \frac{h}{2}$$



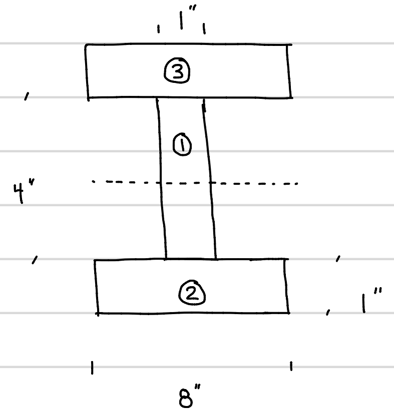
1st moment of area (graphically) $bh \cdot \frac{h}{2} = \frac{bh^2}{2}$

$$I = \int y^2\,dA = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} y^2\,dy\,dx = \int_{-b/2}^{b/2} \frac{h^3}{12}\,dx = \frac{bh^3}{12}$$

about N.A.



origin is located at centroid



Parallel Axis Theorem

$$I_{section} = \sum_{i=1}^{n} I_i + A_i d_i^2$$

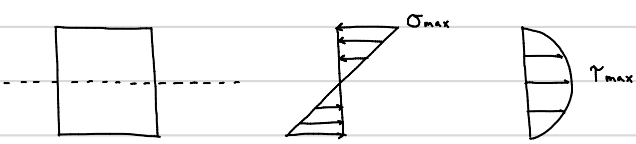
area

distance of element centroid to overall section centroid

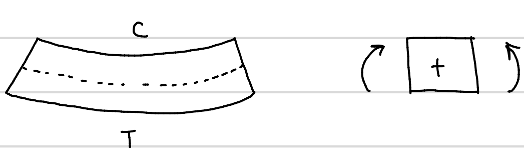
$$I_{section} = \frac{1 * 4^3}{12} + 1 * 4 * 0^2 + 2\left[\frac{8 * 1^3}{12} + 8 * 1 * 2.5^2\right]$$

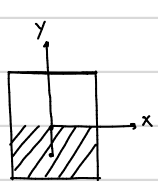
$$= 5.33 + 2\left[0.66 + 50\right] = 106.7 \text{ in.}^4$$

* this is why I-beams are used in structures



$$\sigma = \frac{-My}{I} \qquad \tau = \frac{VQ}{Ib}$$





$$Q_{max} = (b * h/2)\, h/4$$

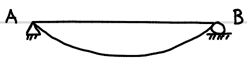
$$\sigma_{max} = \frac{-M_{max}\, y_{max}}{I} = \frac{-M_{max}\, h/2}{\frac{bh^3}{12}} = \frac{-M_{max}}{\frac{bh^2}{6}} \longrightarrow S \equiv \text{section modulus} = \frac{I}{y}$$

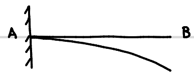
$$\tau_{max} = \frac{V_{max}\, Q_{max}}{I\, b} = \frac{V_{max}\,(b * h/2 * h/4)}{\frac{bh^3}{12} * b} = \frac{V_{max}\, \frac{bh^2}{8}}{\frac{b^2 h^3}{12}} = \frac{3V}{2bh} = \frac{3V}{2A}$$

Deflections (beams)

* Boundary Conditions (B.C.)

"support"





$v_A(x=0)=0$ $v_B(x=L)=0$ $v_A(x=0)=0$

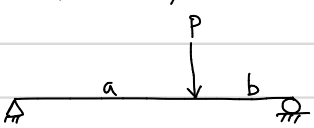
$v_A'(x=0)=0$

$$EIv'' = M(x) \quad \text{moment-curvature}$$
$$EIv' = \int M(x)$$
$$EIv = \iint M(x)$$

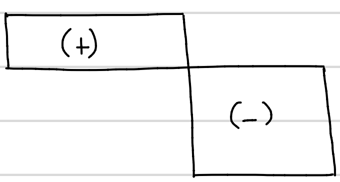
How do we find constants of integration?

i.e., $\int x\,dx = \frac{x^2}{2} + C$

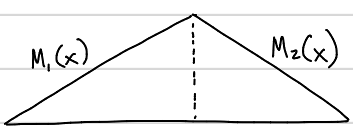
* Compatibility (or continuity) Conditions

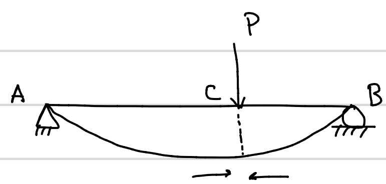


$V(x)$



$M(x)$



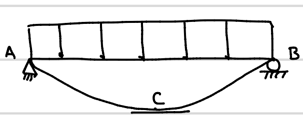


$v_{AC} = v_{CB}$

$v'_{AC} = v'_{CB}$

* Boundary + Compatibility conditions ALWAYS sufficient to determine constants of integration

Symmetry Conditions - useful in some special cases, but NOT required



B.C.s $v_A = v_B = 0$

$v_A' = -v_B'$ $v_C' = 0$